Quark and Lepton Masses from Top Loops

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Work with Paddy Fox

Many attempts at explaining the hierarchy of standard model Yukawa couplings:

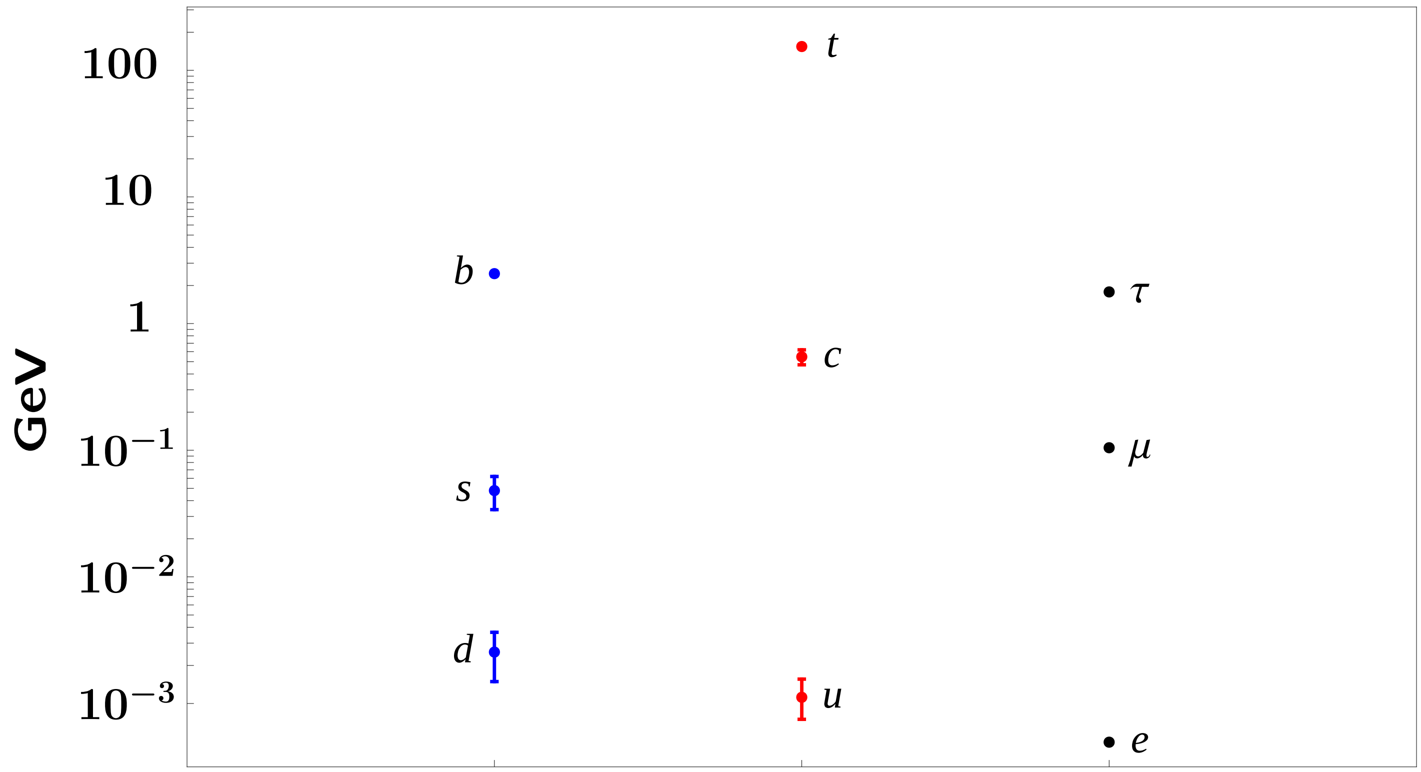
- **discrete symmetries** $\rightarrow (\langle\phi\rangle/M)^n$ **suppressions.**
- **GUT relations.**
- **wave function overlaps in extra dimensions.**
- **...**
- **loop suppressions:**

  Georgi, Glashow, 1972 – **attempts to calculate the electron mass as a one loop contribution involving the muon mass.**

  Many papers in the 1980's (*e.g., Balakrishna, Kagan, Mohapatra, 1988)*

  Typical scheme: *3rd generation masses at tree level,*
  2nd generation masses at one loop,
  1st generation masses at two loops.

However, the fermion masses (at ~ 1 TeV) look more complicated:



Let us assume that only the top quark gets its mass at tree level, $y_t \bar{t}_R Q_L^3 H$, and introduce some interactions that communicate EWSB to the other quarks and leptons.

r: scalar field transforming as (3,2,+7/6) under
$SU(3)_c \times SU(2)_W \times U(1)_Y$

$$r = \begin{pmatrix} r_u \\ r_d \end{pmatrix} \quad \begin{array}{l} \text{charge} + 5/3 \\ \text{charge} + 2/3 \end{array}$$

Most general renormalizable interactions with SM fermions

$$\lambda_{ij}\, r\, \overline{u}^i_R L^j_L + \lambda'_{ij}\, r\, \overline{Q}^i_L e^j_R \qquad (r \text{ is a leptoquark})$$

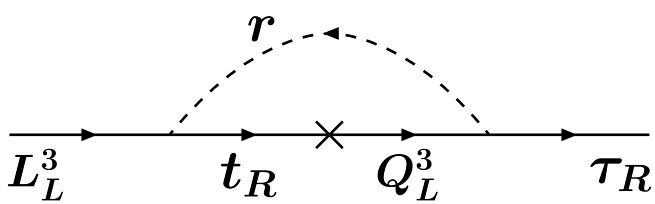
break explicitly the chiral symmetries of the

quarks $U(2)_Q \times U(2)_u \times U(1)_t \times U(3)_d \; \rightarrow \; U(1)_u \times U(3)_d$

and leptons $U(3)_L \times U(3)_e \rightarrow U(1)_L$

$\Rightarrow$ all up-type quarks and electrically-charged leptons get masses at some loop level.

The 1-loop diagram responsible for the tau mass:



Only one linear combination of L_L^j couples to t_R
$\Rightarrow m_\mu = m_e = 0$ at 1-loop

Only one linear combination of e_R^j couples to Q_L^3

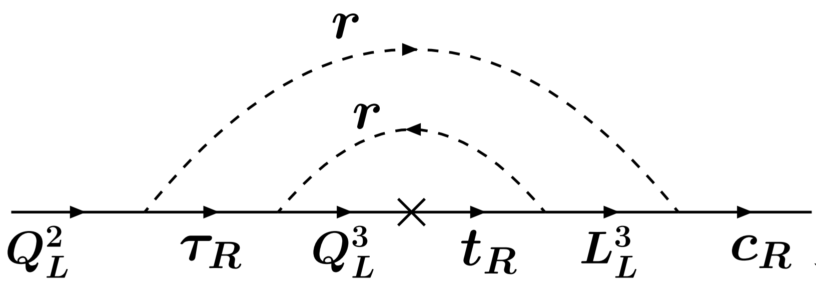
$$m_\tau \simeq \lambda_{33}\lambda'_{33}\, m_t \frac{N_c}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_r^2}\right)$$

Some new physics cuts off the loop integral at a scale Λ:

a superpartner of r, or some dynamics if r is a composite particle, or some particle integrated out to generate the Yukawa couplings of r.

m_τ *depends on* $\frac{\Lambda}{M_r}$ *(only a lower limit on M_r is set by phenomenology).*

Charm mass induced by a 2-loop "rainbow" diagram:



Only one linear combination of Q_L^1 and Q_L^2 couples to $\tau_R \Rightarrow m_u = 0$ at 1-loop

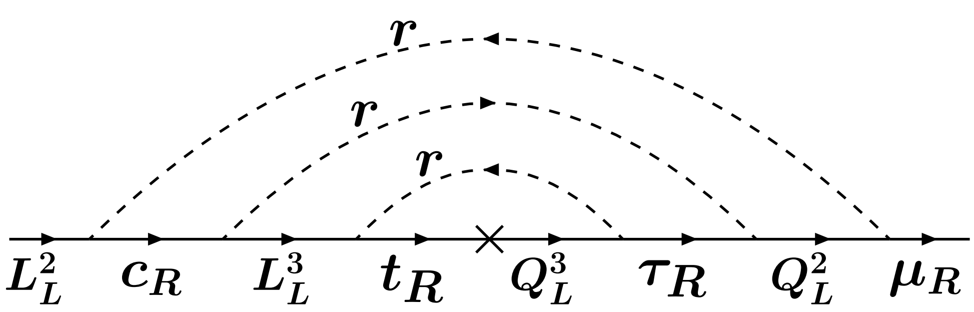
Only one linear combination of u_R^1 and u_R^2 couples to L_L^3

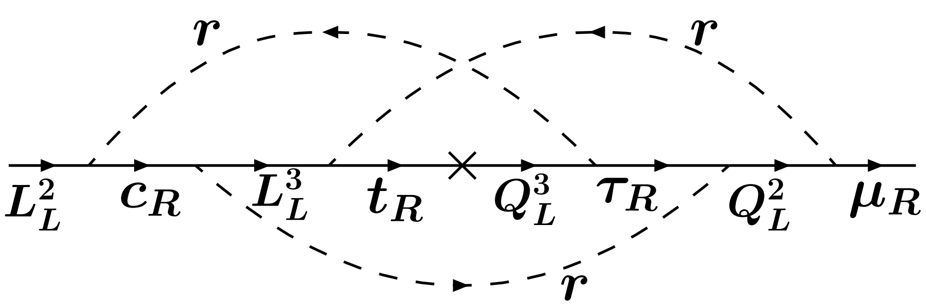
$$m_c \simeq \lambda'_{23}\lambda_{23}\, m_\tau \frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_r^2}\right)$$

If there are no other contributions to m_c,

the m_c/m_τ *ratio at 1 TeV requires* $\lambda_{23}\lambda'_{23} \approx (3.3)^2$ *for* $\Lambda \approx 10 M_r$.

Muon mass induced by 3-loop "rainbow" and nonplanar diagrams:



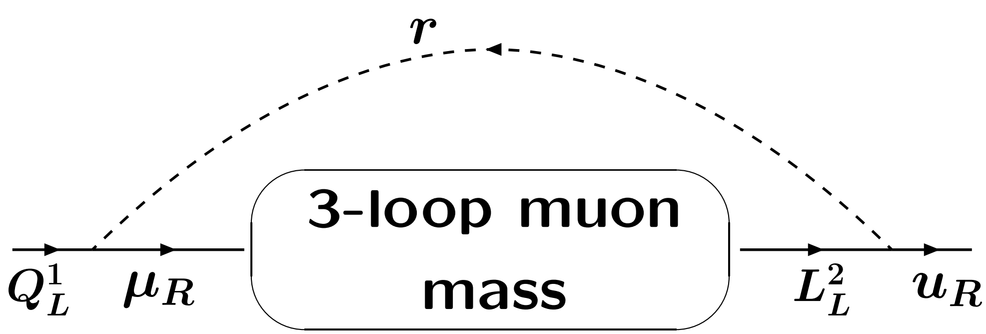


$$m_\mu \simeq \lambda'_{22}\lambda_{22}\, m_c \,[1 + O(1/N_c)]\, \frac{N_c}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_r^2}\right)$$

m_μ/m_c **ratio requires** $\lambda_{22}\lambda'_{22}\,[1 + O(1/N_c)] \approx (1.5)^2$

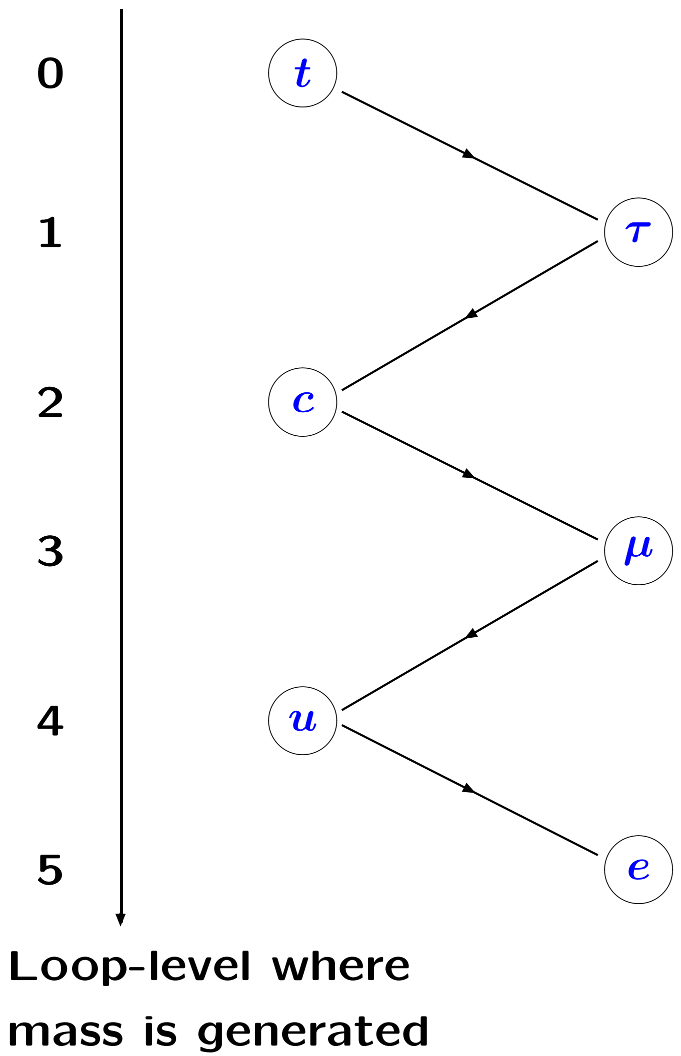
At 3-loops the electron is still massless!

Up-quark mass induced at 4-loops:

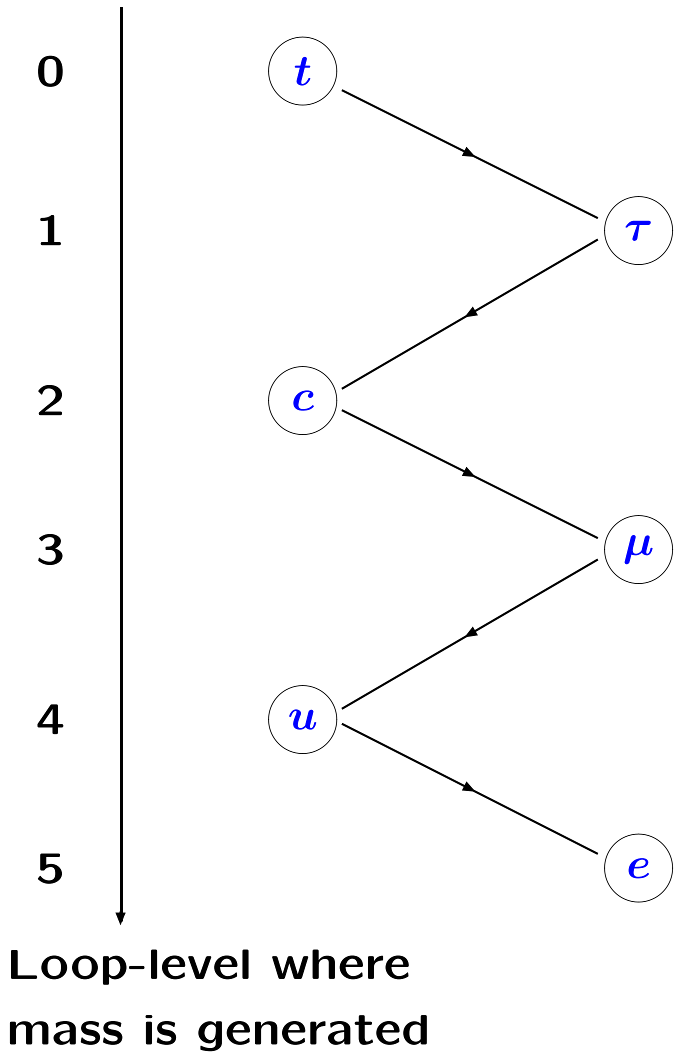


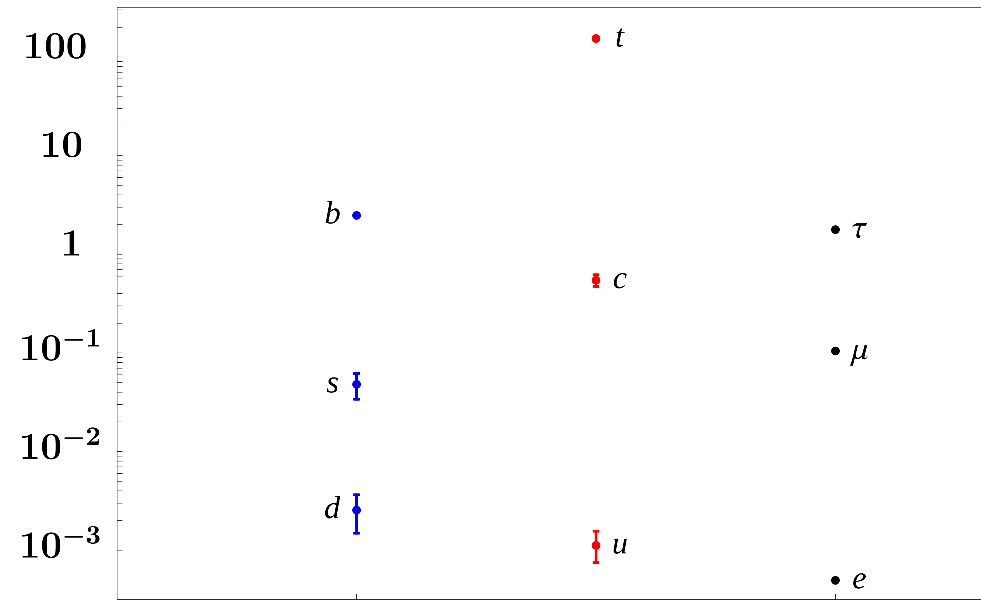
$$m_u \simeq \lambda'_{12}\lambda_{12}\, m_\mu \frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_r^2}\right)$$

Correct m_u/m_μ ratio requires $\lambda_{12}\lambda'_{12} \approx (0.6)^2$



"Domino" mechanism





Only input: $\lambda_{ij}\, r\, \overline{u}_R^i L_L^j + \lambda'_{ij}\, r\, \overline{Q}_L^i e_R^j$ and m_t.

$$\lambda \sim \lambda' \sim \begin{pmatrix} 2.3 & 0.6 & 0 \\ 0 & 1.5 & 3.3 \\ 0 & 0 & 0.4 \end{pmatrix}$$

"Domino" mechanism

Physics at the "cut-off" scale

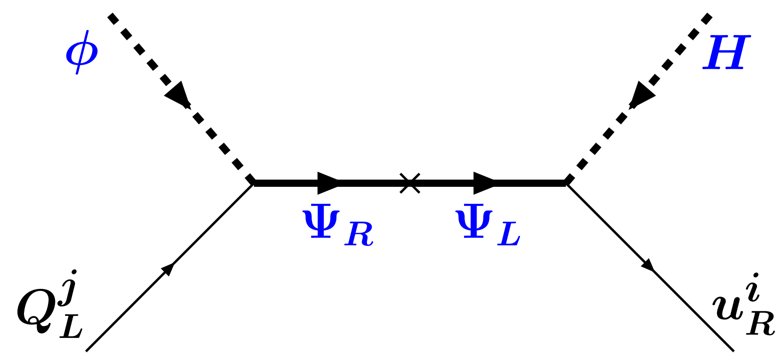
Assume:

- **Higgs doublet is charged under some new symmetry $\mathcal{G}_H$, which forbids dimension-4 couplings to standard model fermions.**

- **$\mathcal{G}_H$ is broken by the VEV of a scalar ϕ which is a singlet under $SU(3) \times SU(2) \times U(1)$.**

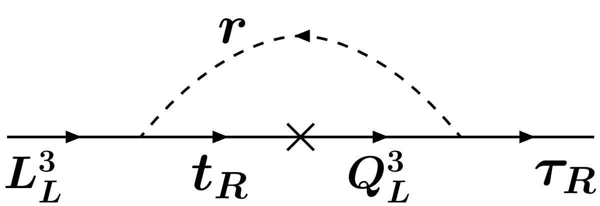
Introduce a vectorlike fermion, Ψ, having same gauge charges as Q_L.

Field redefinitions of u_R^i and Q_L^j
$\Rightarrow$ only one quark couples
to H at tree level: $\frac{1}{M_\Psi}\phi H \, \overline{u}_R^3 Q_L^3$
$\Rightarrow$ effective top Yukawa coupling.

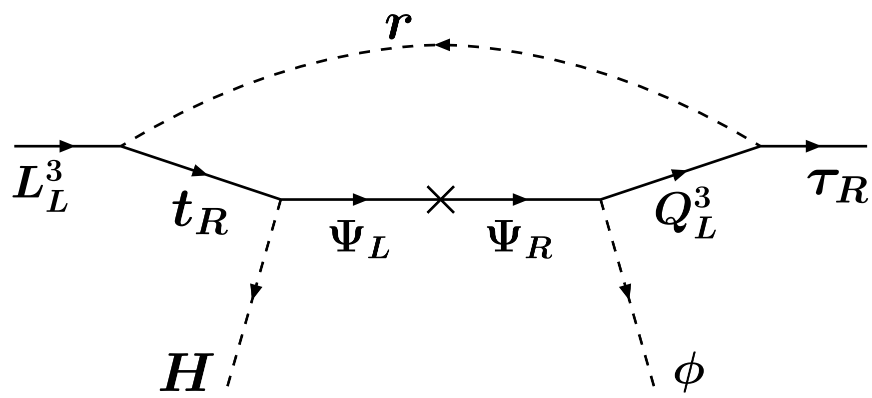


Only the top quark acquires mass at tree level even when no symmetry differentiates it from other standard model fermions!

Loop-induced masses are finite contributions to the coefficient of a dimension-5 operator times $\langle\phi\rangle\langle H\rangle$:



$\Rightarrow$



$$m_\tau \simeq \lambda_{33}\lambda'_{33}\frac{N_c m_t}{16\pi^2}\ln\left(\frac{\Lambda^2}{M_r^2}\right)$$

Coefficient of $\phi H \overline{\tau}_R L_L^3$ operator:

$y_H y_\phi \lambda_{33}\lambda'_{33}\, N_c$ times a finite integral

$$M_\Psi \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2\left(k^2 - M_\Psi^2\right)\left(k^2 - M_r^2\right)} = \frac{1}{16\pi^2}\,\frac{M_\Psi}{M_\Psi^2 - M_r^2}\ln\left(\frac{M_\Psi^2}{M_r^2}\right)$$

$m_t \simeq y_H y_\phi \frac{1}{M_\Psi}\langle\phi\rangle\langle H\rangle \qquad \Rightarrow \qquad \Lambda \simeq M_\Psi$ for $M_\Psi \gg M_r$

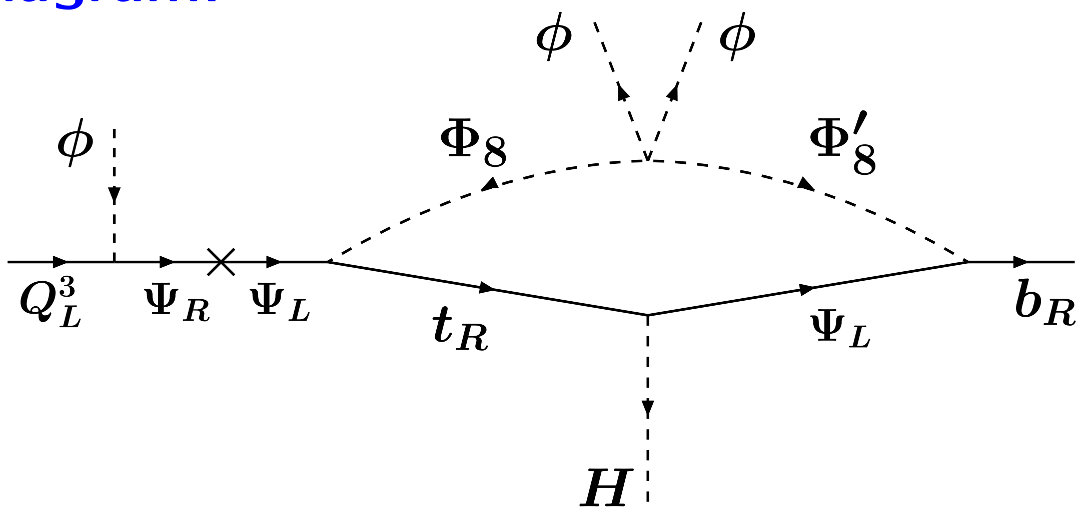
Down-type quark masses

Φ_8, Φ'_8: scalar fields with charges $(8, 2, \pm 1/2)$
under $SU(3)_c \times SU(2)_W \times U(1)_Y$, and $+1$ under $\mathcal{G}_H$.

Couplings to quarks: $\kappa_i \, \Phi_8 \, \overline{u}^i_R \Psi_L + \kappa' \, \Phi'_8 \, \overline{d}^3_R \Psi_L$ (only b_R couples!)

Quartic scalar coupling: $\Phi_8 \Phi'_8 \phi\phi$

m_b induced by a 1-loop diagram:

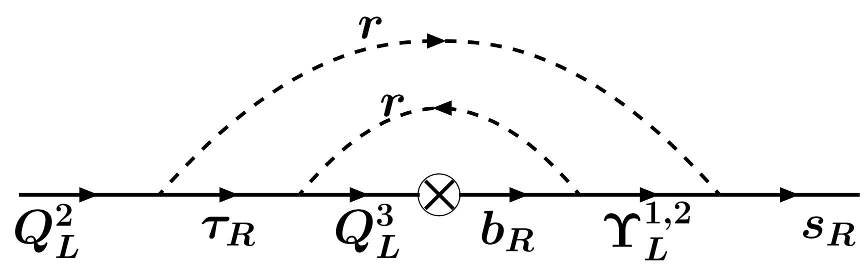


$U(2)$ chiral symmetry of $d_R^{1,2}$ has not been broken so far.

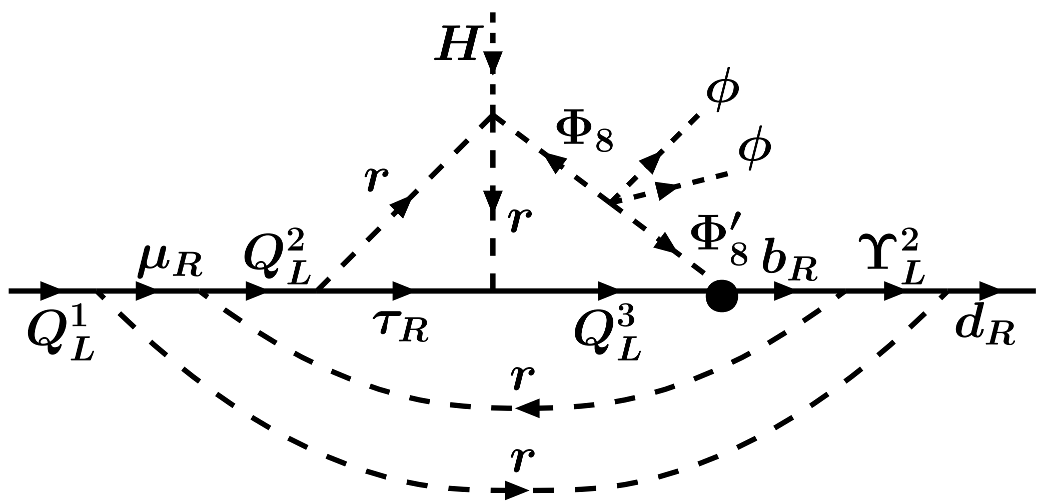
$\rightarrow$ a new particle must couple to $d_R^{1,2}$ in order to generate m_s & m_d

$\Upsilon_{L,R}$: fermion of charges $(1, 2, +3/2)$ under $SU(3)_c \times SU(2)_W \times U(1)_Y$, and 0 under $\mathcal{G}_H$.

m_s induced by a 3-loop diagram:



m_d induced at 4 loops:



Summary of field content:

	H	ϕ	$\Psi_{L,R}$	r	Φ_8	Φ'_8	$\Upsilon^{1,2}_{L,R}$
$SU(3)_c$	1	1	3	3	8	8	1
$SU(2)_W$	2	1	2	2	2	2	2
$U(1)_Y$	$+1/2$	0	$+1/6$	$+7/6$	$+1/2$	$-1/2$	$+3/2$
$\mathcal{G}_H$	$+1$	-1	-1	0	$+1$	$+1$	0
spin	0	0	1/2	0	0	0	1/2

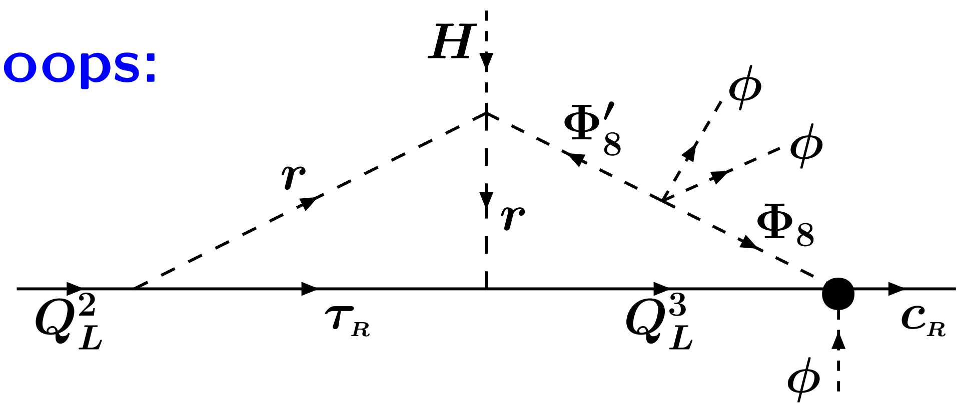
give up-type quark and charged lepton masses

give down-type quark masses.

$SO(10)$ **GUT:** $16 \cdot 16 \cdot \overline{126}$

$$\overline{126} \supset \phi + r + \Phi_8 + \Phi'_8 + H + \ldots$$

Additional m_c induced at 2 loops:



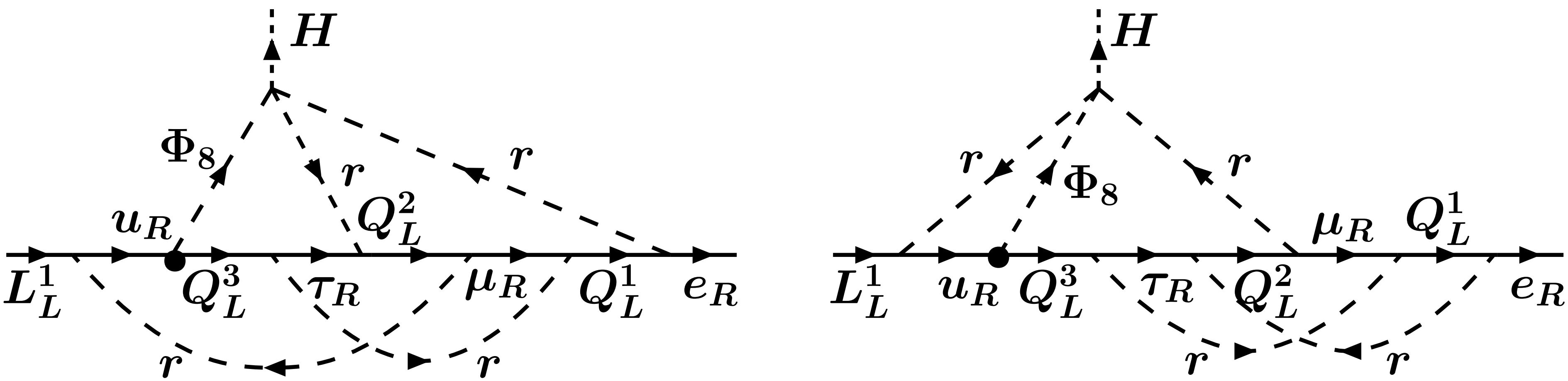
$$M_u[\Phi_8 r] = \begin{pmatrix} 0 & \kappa_1\lambda'_{23} & \kappa_1\lambda'_{33} \\ 0 & \kappa_2\lambda'_{23} & \kappa_2\lambda'_{33} \\ 0 & \kappa_3\lambda'_{23} & \kappa_3\lambda'_{33} \end{pmatrix} \lambda'_{33} c' \, \frac{y_\phi \, \langle\phi\rangle \, v_H}{M_\Psi} \, \epsilon_\Phi^{(2)}$$

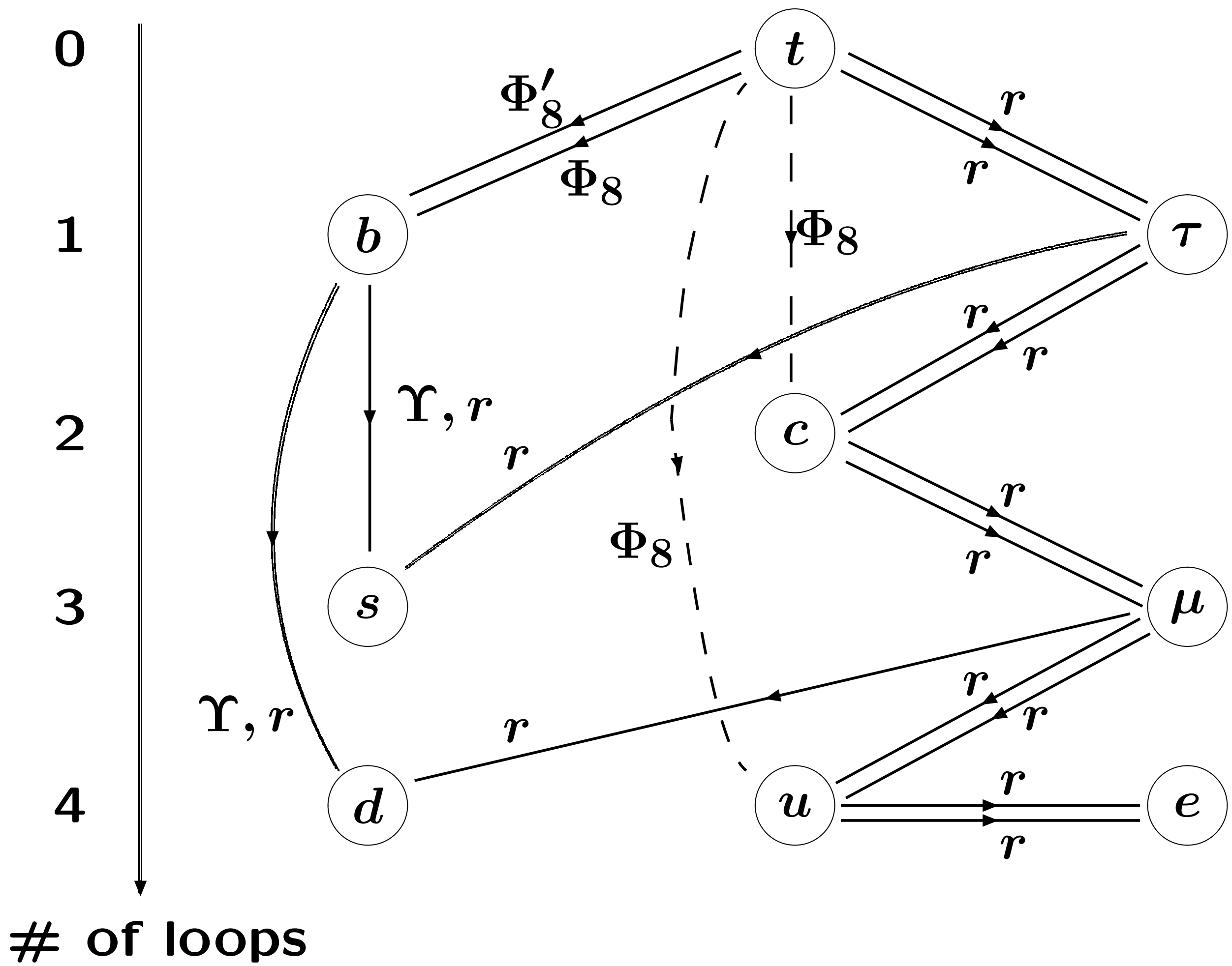
$$\begin{aligned} \epsilon_\Phi^{(2)} &= N_c \int \frac{d^4k'}{(2\pi)^4} \frac{M_\Psi^2 \, \not{k}'}{k'^2(k'^2 - M_8^2)(k'^2 - M_\Psi^2)} \int \frac{d^4k}{(2\pi)^4} \frac{\not{k}}{k^2(k^2 - M_r^2)\left[(k-k')^2 - M_r^2\right]} \\ &= N_c \frac{M_\Psi^2}{16\pi^2} \int_0^1 dx \int_0^1 dy \; \widetilde{I}_1\left(M_\Psi, M_8, M_r\sqrt{(1/x - y)/(1-x)}\right) \end{aligned}$$

$$\widetilde{I}_1(M_\Psi, M_8, m) = \frac{m^2\, M_8^2 \ln(m/M_8) + M_\Psi^2\, m^2 \ln(M_\Psi/m) + M_8^2\, M_\Psi^2 \ln(M_8/M_\Psi)}{8\pi^2\,(m^2 - M_8^2)\,(M_\Psi^2 - M_8^2)\,(M_\Psi^2 - m^2)}$$

For $M_8 \ll M_r, M_\Psi$, and expanding in $M_r^2/M_\Psi^2 \ll 1$: $\quad \epsilon_\Phi^{(2)} \approx \dfrac{N_c}{(16\pi^2)^2}\left[\ln\left(\dfrac{M_\Psi^2}{M_r^2}\right) - \dfrac{\pi^2}{6}\right]$

Additional contributions to the electron mass from non-planar 4-loop diagrams:





Each line connecting a pair of fermions indicate interactions that break their chiral symmetries.

Quark mass matrices:

$$M_u \approx m_t \begin{pmatrix} \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} \qquad M_d \approx m_t \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon \end{pmatrix}$$

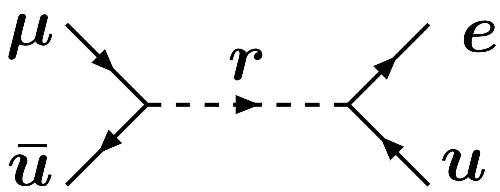
$$\epsilon \approx \frac{N_c}{16\pi^2} \ln \frac{M_1^2}{M_2^2} \approx O(0.1)$$

$$\Rightarrow \quad V_{CKM} = R_u^\dagger R_d \approx \begin{pmatrix} 1-\epsilon^2 & \epsilon & \epsilon^3 \\ -\epsilon & 1-\epsilon^2 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

Phenomenological constraints

Tree level flavor-changing processes induced by r exchange:

$\mu \to e$ **conversion**



$K^+ \to \pi^0 \mu^+ e^-$, ...

$\tau^+ \to K^0 e^+$, ...

$\pi^+ \to e^+ \nu$ **versus** $\pi^+ \to \mu^+ \nu$

...

$\Rightarrow \quad M_r > O(100)$ **TeV.**

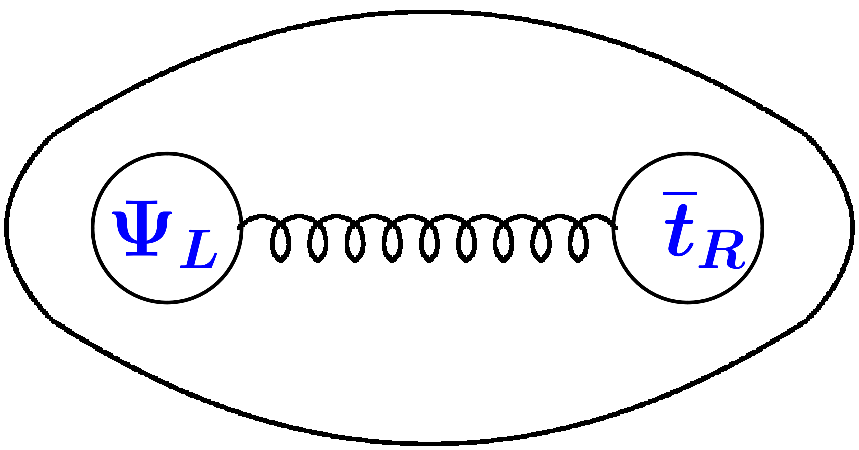
Top condensation $\Rightarrow$ Higgs boson is a $\bar{t}t$ bound state!

(Nambu; Miransky, Tanabashi, Yamawaki; Bardeen, Hill, Lindner, ...)

Binding may be due to some strongly-interacting heavy gauge bosons

New heavy quarks (vectorlike) could bring scale of Higgs compositeness down to a few TeV.

Explicit models: top seesaw, QCD in extra dimensions, ...



Conclusions

• If EWSB is communicated from the top quark to the other fermions by couplings to some new scalar fields, then realistic mass spectra are induced by multi-loop contributions.

• Searches for new flavor-changing processes may unravel the origin of quark and lepton masses.

Work done with Paddy Fox at Fermilab.